

SI 2013 Mark Scheme Q1

(i) $y = \sqrt{x} \Rightarrow y^2 + 3y - \frac{1}{2} = 0$ $(2y + 3)^2 = 11$	B1 for a correct quadratic eqn. in y or $\sqrt{x}$ M1 for use of a method for solving a quadratic eqn. (compl^g. the square, formula, etc.) If candidate fails to obtain a numerical answer for y (correct or not) then M0
$y = \frac{-3 \pm \sqrt{11}}{2}$	A1
$y \geq 0 \Rightarrow \sqrt{x} = \frac{\sqrt{11} - 3}{2}$	M1 for clearly choosing the correct root: FT <i>provided</i> they have 1 +_{ve} and 1 -_{ve} root to choose from
$x = \left(\frac{\sqrt{11} - 3}{2} \right)^2$ or $\frac{20 - 6\sqrt{11}}{4}$ or $5 - \frac{3}{2}\sqrt{11}$	A1

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(ii) (a) $y = \sqrt{x + 2}$ $y^2 + 10y - 24 = 0$	M1 for clear indication of this substitution (or equivalent) A1 for a correct quadratic
y or $\sqrt{x + 2} = -12, 2$ $y \geq 0 \Rightarrow \sqrt{x + 2} = 2$	M1 for solution method of a suitable quadratic M1 for choosing valid root: FT <i>provided</i> they have 1 +_{ve} and 1 -_{ve} root to choose from
$x = 2$	A1

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(ii) (b) $y = \sqrt{2x^2 - 8x - 3}$ $y^2 + 2y - 15 = 0$	M1 for clear indication of this substitution (or equivalent) A1 for a correct quadratic
y or $\sqrt{2x^2 - 8x - 3} = -5, 3$ $y \geq 0 \Rightarrow \sqrt{2x^2 - 8x - 3} = 3$	M1 for solution of a suitable quadratic M1 for choosing valid root: FT <i>provided</i> they have 1 +_{ve} and 1 -_{ve} root to choose from
$2x^2 - 8x - 3 = 9 \Rightarrow x^2 - 4x - 6 = 0$ $x = 2 \pm \sqrt{10}$	M1A1 for obtaining <u>and solving</u> a quadratic eqn. in x; A1 for the correct quadratic
	A1

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$$x = 2 \pm \sqrt{10} \Rightarrow x^2 = 14 \pm 4\sqrt{10}$$

M1 for checking attempt (for at least one of the answers found)

$$\text{so } x^2 - 4x - 9 = -3 \text{ \& } 2x^2 - 8x - 3 = 9$$

$$\Rightarrow (\text{both cases}) -3 + \sqrt{9} = 0$$

A1A1 one for each clearly shown (with working)

ALTERNATIVELY For validity, $2x^2 - 8x - 3 \geq 0$ also **M1** i.e. $(x-2)^2 \geq \frac{11}{2}$ **A1**

Since $(x-2)^2 = 10 > \frac{11}{2}$ both solns. valid **E1**

ALTERNATIVES

(i) $3\sqrt{x} = \frac{1}{2} - x$ and squaring **M1** $x^2 - 10x + \frac{1}{4} = 0$ **A1** correct quadratic **M1** for solution of a suitable quadratic **A1** $x = 5 \pm \frac{3}{2}\sqrt{11}$
However, *both* these roots are positive, so the final mark will be **E1** for checking both, with working, and correctly discarding the unsuitable answer

(ii) (a) $10\sqrt{x+2} = 22 - x$ and squaring **M1** $x^2 - 144x + 284 = 0$ **A1** correct quadratic **M1** for solution of a suitable quadratic **A1** $x = 142, 2$
E1 for checking both, with working, and correctly discarding the unsuitable answer (e.g. $x = 142$ gives LHS > 0 but RHS < 0 would suffice)

(ii) (b) $\sqrt{2x^2 - 8x - 3} = 9 + 4x - x^2$ and squaring **M1** $x^4 - 8x^3 - 4x^2 + 80x + 84 = 0$ **A1** correct quartic
 $(x-2)^4 - 28(x-2)^2 + 180 = 0$ **M1A1** $\Rightarrow (x-2)^2 = 10, 18$ **M1A1**

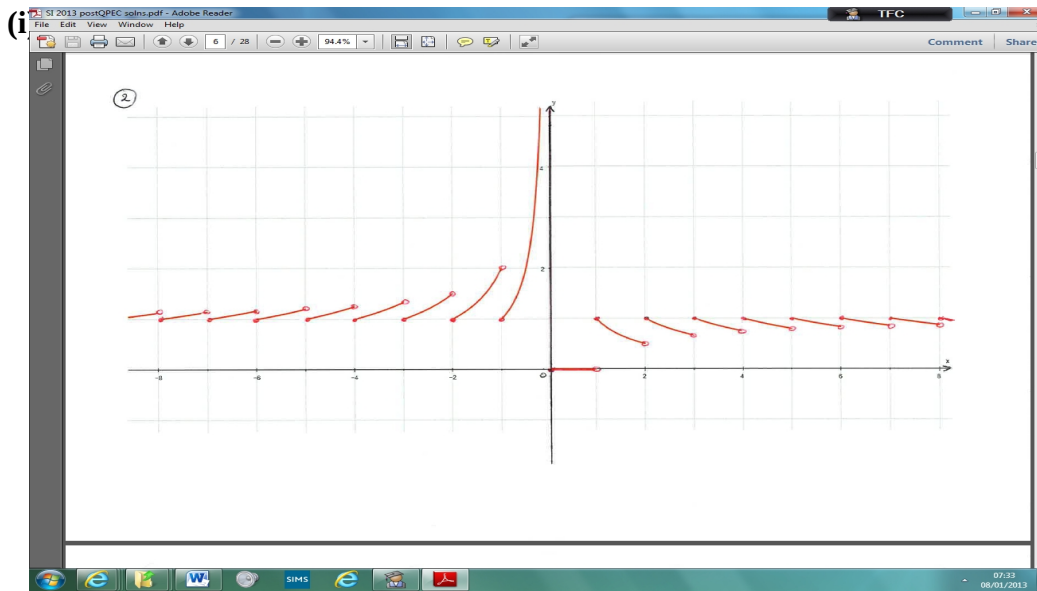
Now $\sqrt{2}\sqrt{(x-2)^2 - \frac{11}{2}} = 13 - (x-2)^2 \Rightarrow \frac{11}{2} \leq (x-2)^2 \leq 13$ **M1A1A1** so the only valid solutions arise from $(x-2)^2 = 10$ and $x = 2 \pm \sqrt{10}$ **A1**

However, I cannot see candidates making this approach work. **M1A1** for getting the correct quartic may be all they can reasonably get. Attempts to find linear factors (by the *Factor Theorem*, for instance) will go nowhere.

Some may attempt to find a pair of quadratic factors: $(x^2 + Ax + B)(x^2 + Cx + D) \equiv x^4 + (A+C)x^3 + (AC+B+D)x^2 + (AD+BC)x + BD = 0$ and compare terms ($A+C=-8$, $AC+B+D=-4$, $AD+BC=80$ and $BD=84$) but I do not want them to have any marks unless they can get to (by guessing/verifying ... divine inspiration?) $(x^2 - 4x - 6)(x^2 - 4x - 14)$, at which point I would award them the next **M1A1** & **M1A1**.

They now have four answers to check for and I would propose a **B1** for each correctly checked (with working) and accepted/rejected appropriately.

SI 2013 Mark Scheme Q2



G1 Lots of “unit” segments

G1 $\left. \begin{array}{l} \text{LH} \\ \text{RH} \end{array} \right\} \text{end clearly} \left\{ \begin{array}{l} \text{included} \\ \text{excluded} \end{array} \right.$

G1 Each segment looks like a portion of a reciprocal curve

G1 Essentially correct $0 \leq x \leq 3$

G1 Essentially correct $-3 \leq x < 0$

Ignore endpoint issues for these last two Gs

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(ii) Note that, for $n \leq x < n+1$, $\lfloor x \rfloor = n$ so $f(x) = \frac{n}{x}$. Also, $\frac{n}{n+1} < f(x) \leq 1$ for $x > 0$, and $f(x) \geq 1$ for $x < 0$, so ...

$f(x) = \frac{7}{12}$ only in $[1, 2)$.

E1 Sketch may show it so

$$f(x) = \frac{1}{x} = \frac{7}{12} \Rightarrow x = \frac{12}{7}$$

B1 B0 if extra answers appear

2

Similarly, $\frac{n}{n+1} > \frac{17}{24} \Rightarrow 24n > 17n + 17 \Rightarrow n > 2\frac{3}{7}$, i.e. $n \geq 3$; so $f(x) = \frac{17}{24}$ only in $[1, 2)$ and $[2, 3)$.

$$\text{In } [1, 2), \quad f(x) = \frac{1}{x} = \frac{17}{24} \Rightarrow x = \frac{24}{17} \quad \text{B1}$$

$$\text{In } [2, 3), \quad f(x) = \frac{2}{x} = \frac{17}{24} \Rightarrow x = \frac{48}{17} \quad \text{B1}$$

Give max. B1 if extra answers appear

2

Now, for $x < 0$, $1 \leq f(x) < \frac{n}{n+1}$, and $\frac{-n}{-n-1} < \frac{4}{3} \Rightarrow -4n - 4 > -3n \Rightarrow n < -4$; so $f(x) = \frac{4}{3}$ only in $[-4, -3)$, $[-3, -2)$, $[-2, -1)$ and $[-1, 0)$.

$f(-3) = 1$ so no solution in $[-4, -3)$ **B1** Possibly implicitly, if just not there

$$\text{In } [-3, -2), \quad f(x) = \frac{-3}{x} = \frac{4}{3} \Rightarrow x = -\frac{9}{4} \quad \text{B1}$$

$$\text{In } [-2, -1), \quad f(x) = \frac{-2}{x} = \frac{4}{3} \Rightarrow x = -\frac{3}{2} \quad \text{B1}$$

$$\text{In } [-1, 0), \quad f(x) = \frac{-1}{x} = \frac{4}{3} \Rightarrow x = -\frac{3}{4} \quad \text{B1}$$

Give max. B1B1 if extra answers appear

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(iii) $\frac{n}{n+1} > \frac{9}{10}$ for $n > 9$ so ...

$$f(x_{\max}) = \frac{9}{10} \text{ in } [8, 9) \quad \text{E1}$$

$$\text{and } f(x) = \frac{8}{x} = \frac{9}{10} \Rightarrow x = \frac{80}{9} \quad \text{B1}$$

NB $f(10) = 1$, so $x \neq 10$

2

$f(x) = c$ has exactly n roots for ...

$$\text{for } x > 0: \quad \frac{n}{n+1} < c \leq \frac{n+1}{n+2} \quad \text{B1B1 LHS; RHS}$$

$$\text{for } x < 0: \quad \frac{n+1}{n} \leq c < \frac{n}{n-1}, \quad n \geq 2 \quad \text{B1B1 LHS; RHS}$$

$$c \geq 2, \quad n = 1 \quad \text{B1}$$

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SI 2013 Mark Scheme Q3

(i)	$X * Y = Y * X \Leftrightarrow \lambda \mathbf{x} + (1 - \lambda) \mathbf{y} = \lambda \mathbf{y} + (1 - \lambda) \mathbf{x}$ $\Leftrightarrow (2\lambda - 1)(\mathbf{x} - \mathbf{y}) = \mathbf{0}$ (Since $\mathbf{x} \neq \mathbf{y}$) $\lambda = \frac{1}{2}$	M1 M1 A1	Including correct $Y * X = \lambda \mathbf{y} + (1 - \lambda) \mathbf{x}$
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(ii)	$(X * Y) * Z = \lambda (\lambda \mathbf{x} + (1 - \lambda) \mathbf{y}) + (1 - \lambda) \mathbf{z}$ $= \lambda^2 \mathbf{x} + \lambda(1 - \lambda) \mathbf{y} + (1 - \lambda) \mathbf{z}$ and $X * (Y * Z) = \lambda \mathbf{x} + (1 - \lambda) [\lambda \mathbf{y} + (1 - \lambda) \mathbf{z}]$ $= \lambda \mathbf{x} + \lambda(1 - \lambda) \mathbf{y} + (1 - \lambda) 2 \mathbf{z}$	M1 A1 M1 A1
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$(X * Y) * Z - X * (Y * Z) = \lambda(1 - \lambda)(\mathbf{x} - \mathbf{z})$ The two are distinct provided $\lambda \neq 0, 1$ or $X \neq Z$	M1 A1
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(ii)	$(X * Y) * Z = \lambda^2 \mathbf{x} + \lambda(1 - \lambda) \mathbf{y} + (1 - \lambda) \mathbf{z}$ $(X * Z) * (Y * Z) = [\lambda \mathbf{x} + (1 - \lambda) \mathbf{z}] * [\lambda \mathbf{y} + (1 - \lambda) \mathbf{z}]$ $= \lambda^2 \mathbf{x} + \lambda(1 - \lambda) \mathbf{z} + \lambda(1 - \lambda) \mathbf{y} + (1 - \lambda)^2 \mathbf{z}$ $= \lambda^2 \mathbf{x} + \lambda(1 - \lambda) \mathbf{y} + (1 - \lambda) \mathbf{z}$	M1 A1
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(and the two are always equal)

$$X * (Y * Z) = \lambda \mathbf{x} + \lambda(1-\lambda)\mathbf{y} + (1-\lambda)^2 \mathbf{z}$$

$$(X * Y) * (X * Z) = [\lambda \mathbf{x} + (1-\lambda)\mathbf{y}] * [\lambda \mathbf{x} + (1-\lambda)\mathbf{z}] \quad \mathbf{M1}$$

$$= \lambda^2 \mathbf{x} + \lambda(1-\lambda)\mathbf{y} + \lambda(1-\lambda)\mathbf{x} + (1-\lambda)^2 \mathbf{z}$$

$$= \lambda^2 \mathbf{x} + \lambda(1-\lambda)\mathbf{y} + (1-\lambda)\mathbf{z}$$

A1

$$\text{Hence } X * (Y * Z) = (X * Y) * (X * Z)$$

E1

Conclusion must be stated (before or after proof)

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For $0 < \lambda < 1$, P_1 cuts XY in the ratio $(1-\lambda) : \lambda$

B1

$$X \text{ --- } \frac{1-\lambda}{P_1} \text{ --- } \frac{\lambda}{P_1} \text{ --- } Y$$

$$X \text{ --- } \frac{1-\lambda^2}{P_1} \text{ --- } \frac{\lambda^2}{P_2} \text{ --- } Y$$

M1

Iterating

We see that P_n cuts XY in the ratio

$$1 - \lambda^n : \lambda^n$$

A1A1

Any correct form

$$P_{n+1} = P_n * Y = \lambda \mathbf{p}_n + (1-\lambda)\mathbf{y}$$

$$= \lambda \{ (\lambda^n)\mathbf{x} + (1-\lambda^n)\mathbf{y} \} + (1-\lambda)\mathbf{y}$$

$$= \lambda^{n+1}\mathbf{x} + (\lambda - \lambda^{n+1} + 1 - \lambda)\mathbf{y}$$

$$= \lambda^{n+1}\mathbf{x} + (1 - \lambda^{n+1})\mathbf{y}$$

and proof follows by induction

M1A1 for attempt at an inductive proof; fully correct

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SI 2013 Mark Scheme Q4

(i) $\int \tan^n x \cdot \sec^2 x \, dx = \left[\frac{1}{n+1} \tan^{n+1} x \right]$ **M1** May be done via a substn. such as $t = \tan x$ (or by “parts”)

$= \frac{1}{n+1}$ **A1** **ANSWER GIVEN**

2

$\int \sec^n x \cdot \tan x \, dx = \int \sec^{n-1} x \cdot \sec x \tan x \, dx$ **M1** May be done via a substn. such as $s = \sec x$ (or by parts)

$= \left[\frac{1}{n} \sec^n x \right]$ **A1**

$= \frac{(\sqrt{2})^n - 1}{n}$ **A1** **ANSWER GIVEN**

3

(ii) (a) $\int_0^{\pi/4} x \sec^4 x \tan x \, dx = \left[x \cdot \frac{\sec^4 x}{4} \right]_0^{\pi/4} - \int_0^{\pi/4} \frac{\sec^4 x}{4} \, dx$ **M1A1A1** for appropriate use of *parts*; correct (1st, 2nd)

$= \frac{\pi}{4} - \frac{1}{4} J$ where $J = \int_0^{\pi/4} \sec^4 x \, dx$

$$J = \int_0^{\pi/4} \sec^2 x \, dx + \int_0^{\pi/4} \sec^2 x \tan^2 x \, dx$$

$$= \left[\tan x + \frac{1}{3} \tan^3 x \right]_0^{\pi/4}$$

$$= \frac{4}{3}$$

M1 for use of $\sec^2 x = 1 + \tan^2 x$ to split up the integral

A1 A1

NB Limits are ignored until the end, when numerical answers need to appear

Thus $I = \frac{\pi}{4} - \frac{1}{3}$

A1

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(ii) (b) $\int x^2 (\sec^2 x \cdot \tan x) \, dx$

$$= \left[x^2 \cdot \frac{1}{2} \tan^2 x \right] - \int 2x \cdot \frac{1}{2} \tan^2 x \, dx$$

M1 for appropriate splitting and attempted use of *integration by parts*

A1A1

$$= \frac{\pi^2}{32} - \int x (\sec^2 x - 1) \, dx$$

M1 for use of $\tan^2 x = \sec^2 x - 1$

$$= \frac{\pi^2}{32} - K + \int x \, dx \quad \text{where } K = \int_0^{\pi/4} x \sec^2 x \, dx$$

$$K = \left[x \cdot \tan x \right] - \int \tan x \, dx$$

M1

$$= x \tan x - \ln(\sec x)$$

B1 for $\int \tan x \, dx = \ln(\sec x)$

$$= \frac{\pi}{4} - \frac{1}{2} \ln 2$$

A1

Thus $I = \frac{\pi^2}{32} - \left(\frac{\pi}{4} - \frac{1}{2} \ln 2 \right) + \frac{\pi^2}{32}$

$$= \frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2} \ln 2$$

A1

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Note that there are many ways to split these integrals in (ii) into parts.

SI 2013 Mark Scheme Q5

(i) For $k = 0$: $x^2 + 3x + y^2 + y = 0$

$$\left(x + \frac{3}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \left(\frac{1}{2}\sqrt{10}\right)^2$$

giving a CIRCLE

thro' $(0, 0)$, $(0, -1)$ & $(-3, 0)$

M1 for completing the square for both x and y

G1 for a circle drawn

B1B1 passing thro' the origin; other 2 intercepts correct (noted on sketch or separately)

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(ii) For $k = \frac{10}{3}$: $(3x + y)(x + 3y + 3) = 0$

giving a LINE-PAIR

Lines $y = -3x$ & $x + 3y = -3$

1st line thro' O with $-ve$ gradient

2nd line not thro' O with $-ve$ gradient

thro' $(0, -1)$ & $(-3, 0)$

B1 Must be the full thing (no marks for just factorising the given quadratic)

G1 for two (intersecting) lines drawn

Statement of eqns. not actually required

B1

M1

A1 for both stated or noted on sketch

5

(iii) For $k = 2$: $(x + y)^2 + 3x + y = 0$

When $\theta = 45^\circ$,

$$x + y = X\sqrt{2} \quad \text{and} \quad y - x = Y\sqrt{2}$$

$$\Rightarrow x = \frac{X - Y}{\sqrt{2}} \quad \text{and} \quad y = \frac{X + Y}{\sqrt{2}}$$

B1 noted or used anywhere

M1A1

$$(x + y)^2 + 3x + y = 0 \quad \text{becomes ...}$$

$$2X^2 + \frac{3X - 3Y}{\sqrt{2}} + \frac{X + Y}{\sqrt{2}} = 0$$

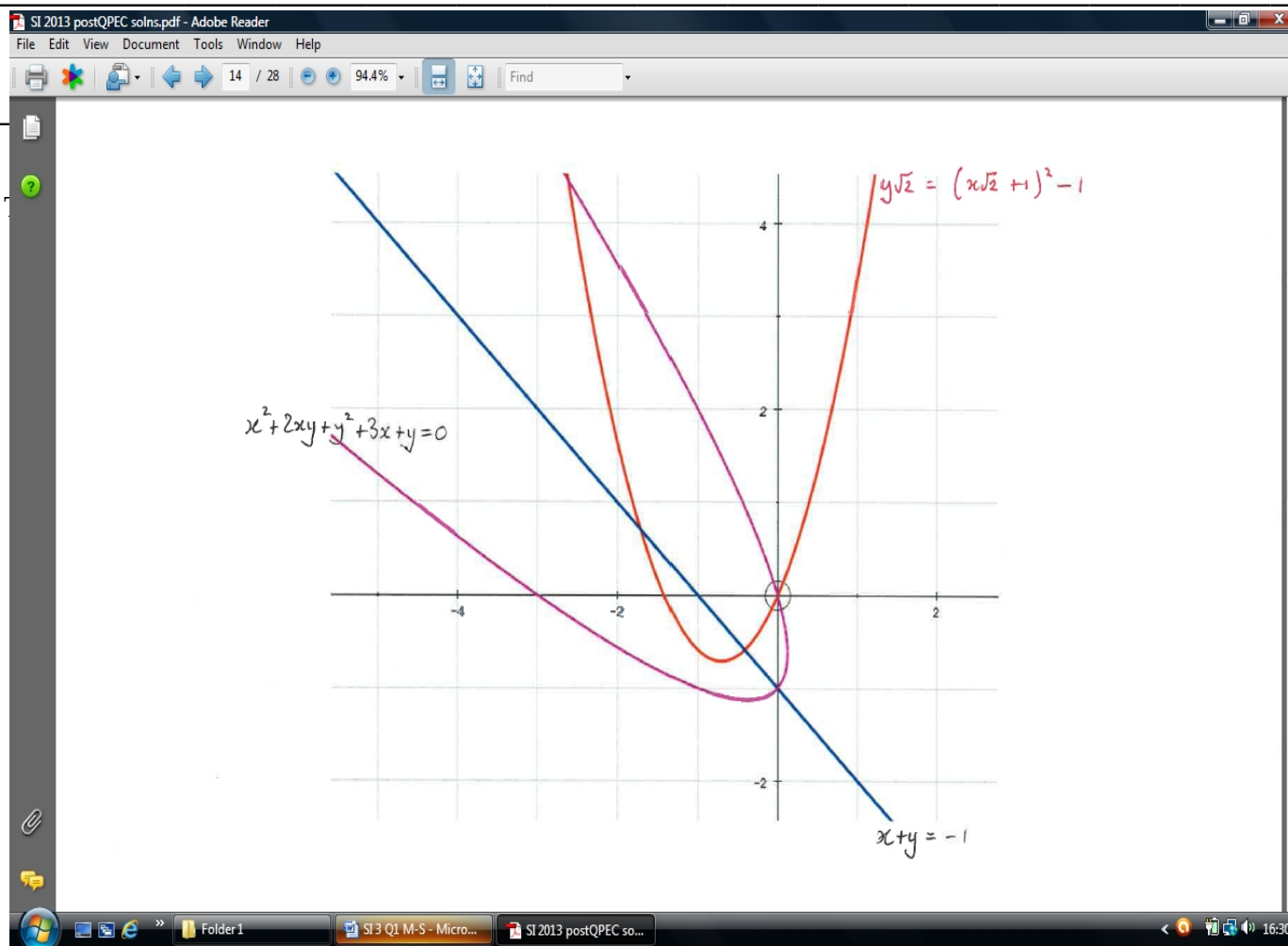
M1 for eliminating both x and y for X and Y

$$\Rightarrow 2X^2 + 2\sqrt{2}X = Y\sqrt{2}$$

$$\Rightarrow (\sqrt{2}X + 1)^2 - 1 = Y\sqrt{2}$$

M1A1 for completing the square; correct **ANSWER GIVEN** obtained

6



l curve correct

rple curve and blue line obviously
otation thro' 45° of their red curve

.L correct (incl. thro' O)

If red curve does not appear ...
 G3 for purple curve & blue line correct
 G1 for obviously rotated 45° c/w
 G0 for anything else

5

SI 2013 Mark Scheme Q6

(*) Coefft. of x^r in $(1+x)^{n+1}$ is $\binom{n+1}{r}$ **B1**

Coefft. of x^r in $(1+x)(1+x)^n$ is from

$$(1+x) \left(\dots + \binom{n}{r-1} x^{r-1} + \binom{n}{r} x^r + \dots \right) \quad \mathbf{M1}$$

$$\Rightarrow \binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$$

A1 **GIVEN ANSWER** legitimately obtained

3

For n even, writing $n = 2m \dots$

M1 for attempting the even case

$$B_{2m} + B_{2m+1} = \binom{2m}{0} + \binom{2m-1}{1} + \binom{2m-2}{2} + \dots + \binom{m+1}{m-1} + \binom{m}{m}$$

B1

$$+ \binom{2m+1}{0} + \binom{2m}{1} + \binom{2m-1}{2} + \binom{2m-2}{3} + \dots + \binom{m+1}{m}$$

B1

$$= \binom{2m+1}{0} + \left[\binom{2m}{0} + \binom{2m}{1} \right] + \left[\binom{2m-1}{1} + \binom{2m-1}{2} \right] + \dots + \left[\binom{m+1}{m-1} + \binom{m+1}{m} \right] + \binom{m}{m} \quad \text{M1 for suitable pairings (clear)}$$

$$= \binom{2m+2}{0} + \left[\binom{2m+1}{1} \right] + \left[\binom{2m}{2} \right] + \dots + \left[\binom{m+2}{m} \right] + \binom{m+1}{m+1} \quad \text{M1 for use of first result, (*)}$$

using the result (*) from above and since $\binom{2m+1}{0} = \binom{2m+2}{0} = \binom{m}{m} = \binom{m+1}{m+1} = 1$ M1 for noting the equality of the "1"s

$$= \sum_{j=0}^{m+1} \binom{2(m+1)-j}{j} = B_{2m+2}$$

A1 Legitimately shown

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For n odd, writing $n = 2m + 1 \dots$

M1 for attempting the odd case

$$B_{2m+1} + B_{2m+2} = \binom{2m+1}{0} + \binom{2m}{1} + \binom{2m-1}{2} + \dots + \binom{m+2}{m-1} + \binom{m+1}{m}$$

Note that this appeared above

$$+ \binom{2m+2}{0} + \binom{2m+1}{1} + \binom{2m}{2} + \binom{2m-1}{3} + \dots + \binom{m+2}{m} + \binom{m+1}{m+1}$$

B1

$$= \binom{2m+2}{0} + \left[\binom{2m+1}{0} + \binom{2m+1}{1} \right] + \left[\binom{2m}{1} + \binom{2m}{2} \right] + \dots + \left[\binom{m+2}{m-1} + \binom{m+2}{m} \right] + \left[\binom{m+1}{m} + \binom{m+1}{m+1} \right]$$

M1 for suitable pairings (clear)

$$= \binom{2m+3}{0} + \left[\binom{2m+2}{1} \right] + \left[\binom{2m+1}{2} \right] + \dots + \left[\binom{m+3}{m} \right] + \binom{m+2}{m+1}$$

M1 for use of first result, (*)

using the result (*) from above and since $\binom{2m+2}{0} = \binom{2m+3}{0} = 1$ M1 for noting the equality of the "1"s

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$$= \sum_{j=0}^{m+1} \binom{2(m+1)+1-j}{j} = B_{2m+3}$$

A1 Legitimately shown

$$B_0, B_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1, 1$$

B1 Evaluating B_0, B_1

Evaluating F_0, F_1 & F_2

B1 Both sides must be evaluated, not just stated

Statement that $B_n = F_{n+1}$

B1 At any point

For a clear justification of result (e.g. inductively)

E1

(i.e. comment that since $B_0 = F_1, B_1 = F_2$ and B_n & F_n satisfy the same recurrence relation, we must have $B_n = F_{n+1}$ for all n)

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SI 2013 Mark Scheme Q7

(i) $y = ux \Rightarrow \frac{dy}{dx} = u + x \frac{du}{dx}$

B1

$$\Rightarrow u + x \frac{du}{dx} = \frac{1}{u} + u$$

M1 substituted in

$$\Rightarrow \int u \, du = \int \frac{1}{x} \, dx$$

M1 variables separated and integration attempted

$$\Rightarrow \frac{1}{2} u^2 = \frac{y^2}{2x^2} = \ln x + C$$

M1 substituted back for x and y

$$\Rightarrow y^2 = x^2(2 \ln x + 2C)$$

$x = 1, y = 2$ substd. to determine C

M1 $C = 2$ (the lack of a constant of integration earlier $\Rightarrow$ M0 here)

$$\Rightarrow y = x\sqrt{2 \ln x + 4}$$

ANSWER GIVEN (all working and signs must be correct throughout)

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E1 Justification of $+_{ve}$ square-root required: $y > 0$ when $x = 1$ gives this
(Given $x > e^{-2}$, so square-rooting valid – this does not need to be stated by candidates)

(ii)	$y = ux$ used to get $u + x \frac{du}{dx} = \frac{1}{u} + 2u$	B1	
	$\Rightarrow \int \frac{2u}{1+u^2} du = \int \frac{2}{x} dx$	M1	variables separated and integration attempted
	$\Rightarrow \ln(1+u^2) = 2 \ln x \quad (+ \ln A)$	A1	
	$x = 1, y = 2$ substd. to determine A	M1	$A = 5$ (the lack of a constant of integration earlier $\Rightarrow$ M0 here)
	$1 + \frac{y^2}{x^2} = 5x^2$	M1	for eliminating u and correct use of log laws (<i>with</i> correct number of terms)
	$y = x\sqrt{5x^2 - 1}$	A1	No need to justify choice of $+_{ve}$ square-root, but don't allow $\pm$
	for $x > \frac{1}{\sqrt{5}}$	B1	FT sensible domains (provided $x > 0$)

(ii) ALTERNATIVE

$y = ux^2$ used to get	$2ux + x^2 \frac{du}{dx} = \frac{1}{ux} + 2ux$	B1M1	$\frac{dy}{dx}$ correct (LHS); full substitution into given d.e.
	$\Rightarrow \int u du = \int \frac{1}{x^3} dx$	M1	variables separated and integration attempted
	$\Rightarrow \frac{1}{2}u^2 = \frac{-1}{2x^2} \quad (+ \frac{1}{2}C)$		
	$x = 1, y = 2$ ($u = 2$) substd. to find C	M1	$C = 15$ (the lack of a constant of integration earlier $\Rightarrow$ M0 here)
	$\frac{y^2}{x^4} = 5 - \frac{1}{x^2}$	M1	for eliminating u
	$y = x\sqrt{5x^2 - 1}$	A1	No need to justify choice of $+_{ve}$ square-root, but don't allow $\pm$
	for $x > \frac{1}{\sqrt{5}}$	B1	FT sensible domains (provided $x > 0$)

(iii)	$y = ux^2$ used to get	$2ux + x^2 \frac{du}{dx} = \frac{1}{u} + 2ux$	B1M1	$\frac{dy}{dx}$ correct (LHS); full substitution into given d.e.
	$\Rightarrow \int u \, du = \int \frac{1}{x^2} \, dx$		M1	variables separated and integration attempted
	$\Rightarrow \frac{1}{2}u^2 = \frac{-1}{x} \quad (+D)$			
	$x = 1, y = 2 \quad (u = 2)$ substd. to find D		M1	$D = 3$ (the lack of a constant of integration earlier $\Rightarrow$ M0 here)
	$\frac{y^2}{x^4} = 6 - \frac{2}{x}$		M1	for eliminating u
	$y = x\sqrt{6x^2 - 2x}$ or $y = x\sqrt{2x}\sqrt{3x-1}$		A1	No need to justify choice of $+_{ve}$ square-root, but don't allow $\pm$
	for $x > \frac{1}{3}$		B1	FT sensible domains (provided $x > 0$)

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SI 2013 Mark Scheme Q8

(i)	function	domain	range	
	$cb(x) = 2 \ln x$	$x > 0$	$\mathbb{R}$	B1
	$ab(x) = (\ln x)^2$	$x > 0$	$y \geq 0$	B1
	$da(x) = x $	$\mathbb{R}$	$y \geq 0$	B1
	$ad(x) = x$	$x \geq 0$	$y \geq 0$	B1
				Do not allow x or $\sqrt{x^2}$ here; but $\begin{cases} x & x > 0 \\ -x & x < 0 \end{cases}$ is ok
				These B1s are for the functions.
				B1B1 1 st for all domains correct; 2 nd for all ranges correct

6

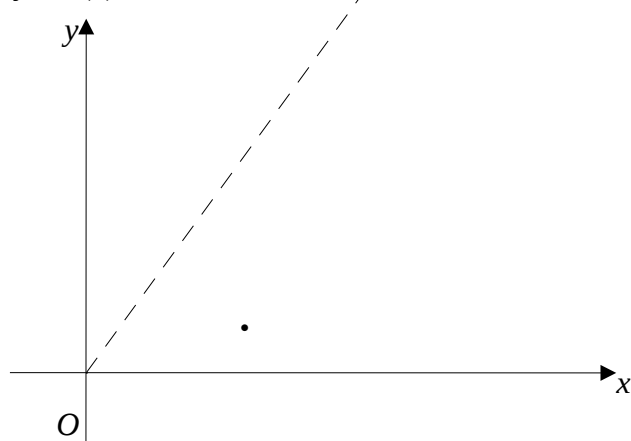
(ii) $fg(x) = |x|$ or $\sqrt{x^2}$ $\mathbb{R}$ $y \geq 0$ **B1 B1** Allow $\begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$ but not or $\pm x$

$gf(x) = |x|$ or $\sqrt{x^2}$ $|x| \geq 1$ $y \geq 1$ **B1 B1**

In each case, 1st B1 for the fn. & 2nd B1 for both domain and range
OR 1st B1 for both domains correct, 2nd B1 for both ranges correct

4

(iii) $y = h(x) = x + \sqrt{x^2 - 1}, x \geq 1$



G1 Starts at (1, 1) – noted or clear on sketch

G1 Going upwards ($\rightarrow \infty$)

G1 Approaching asymptote $y = 2x$ (eqn. must be given somewhere)

NB – I will correct the diagram so it is the graph of a *function*

$y = k(x) = x - \sqrt{x^2 - 1}, |x| \geq 1$

G1 Two branches

G1 (1, 1) and (-1, -1) noted or clear on sketch

G1 Branch in Q1 approaches (from above) asymptote $y = 0$ (or “x-axis”) must be noted somewhere

G1 Branch in Q3 approaches asymptote (from above) $y = 2x$ must be noted somewhere

7

Domain of kh is $x \geq 1$

B1

Range of h is $y \geq 1 \Rightarrow$ Range of kh is $0 < y \leq 1$ **M1A1**
